# The Controlled Teleportation of an Arbitrary Two-Atom Entangled State in Driven Cavity QED

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**Abstract** In this paper, we propose a scheme for the controlled teleportation of an arbitrary two-atom entangled state  $|\phi\rangle_{12} = a|gg\rangle_{12} + b|ge\rangle_{12} + c|eg\rangle_{12} + d|ee\rangle_{12}$  in driven cavity QED. An arbitrary two-atom entangled state can be teleported perfectly with the help of the cooperation of the third side by constructing a three-atom GHZ entangled state as the controlled channel. This scheme does not involve apparent (or direct) Bell-state measurement and is insensitive to the cavity decay and the thermal field. The probability of the success in our scheme is 1.0.

Keywords Controlled teleportation · Bell-state measurement · Driven cavity QED

## 1 Introduction

Since the first scheme for quantum teleportation was proposed by Bennett et al. [1], all kinds of schemes [2–5] have been proposed and experimental demonstration of quantum teleportation has been realized with the polarization photon [6] and a single coherent mode of a field in optical systems [7] and NMR [8]. In 1998, Karlsson and Bourennane [9] showed that an arbitrary unknown state of a qubit could be teleported to either one of two receivers by the use of a three-qubit entangled Greenberger-Horne-Zeilinger (GHZ) state. One of the two agents acts as the controller and the other recovers the unknown state according to the information published by the sender and the controller. Since that work, many investigations have been made on the controlled teleportation [10–12].

Recently, Riebe et al. [13] and Barrett et al. [14] have implemented the first experimental realization of the teleportation of atomic qubits in ion-trap system, which will attract more attention for quantum information processing on the field of cavity QED. In cavity QED, schemes have been proposed for teleportation of two-particle entangled states [15] and multipartite entangled atomic states [16]. However, the main experimental challenge consists in

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the so-called Bell-state measurement. Zheng [17] and Ye [18] proposed a scheme for teleporting an unknown atomic state in cavity QED without Bell-state measurement, but the probability of success is only 0.25 and 0.5. Jin et al. [19] made a proposal for teleporting two-atom entangled state with a probability 1.0 by adding a classical driving field.

As for the arbitrary two-atom two-level entangled state, Lee et al. [20] showed that it was possible to teleport a two-qubit state  $|\phi\rangle_{12} = a|gg\rangle_{12} + b|ge\rangle_{12} + c|eg\rangle_{12} + d|ee\rangle_{12}$  from Alice to Bob using a four-entangled state and sending to him four bits of classical information. Rigolin [21] explicitly constructs this protocol and presents a generalization to N qubits, but this protocol needs a set of 16 generalized Bell states to implement the teleportation. Deng [22] present a way for symmetric multiparty-controlled teleportation of an arbitrary two-particle entangled state based on Bell-basis measurements by using two Greenberger-Horne-Zeilinger states. In this paper, we propose a scheme for the controlled teleportation of an arbitrary two-atom two-level entangled state in driven cavity QED. In contrast to the previous scheme, the present one has the following advantages: First such an arbitrary and unknown two-atom state is transmitted from a sender to a receiver, whereas the scheme in [19] considers a special unknown two-atom state. Second, the quantum channel is different, in our protocol, the quantum channel is composed of the two-atom maximally entangled state comparing with the previous scheme [21] using a four-entangled state. Whereas two-atom maximally entangled state is much easier to prepare and maintain than four-atom entangled state. Finally the protocol does not involve apparent (or direct) Bell-state measurement and is insensitive to the cavity decay and the thermal field. The probability of the success can reach 1.0.

## 2 The Model

Consider two two-level atoms interacting resonantly with a single-mode cavity field, at the same time, the two atoms are driven by a classical field. The interaction between atoms and the cavity can be described as follows [23]:

$$H = \omega_0 \sum_{j=1}^2 S_j^z + \omega_a a^{\dagger} a + \sum_{j=1}^2 [g(a^{\dagger} S_j^- + a S_j^+) + \Omega(S_j^+ e^{-i\omega_d t} + S_j^- e^{i\omega_d t})], \qquad (1)$$

where  $\omega_0$ ,  $\omega_a$  and  $\omega_d$  are atomic transition frequency, cavity frequency and the frequency of driving field, respectively,  $a^{\dagger}$  and a are creation and annihilation operators for the cavity mode, g is the coupling constant between atoms and cavity, atomic operators  $S_j^+ = |e\rangle_j \langle g|, S_j^- = |g\rangle_j \langle e|, S_j^z = \frac{1}{2}(|e\rangle_j \langle e| - |g\rangle_j \langle g|)$ , and  $\Omega$  is the Rabi frequency of the classical field. We consider the case  $\omega_0 = \omega_d$ . In the interaction picture, the evolution operator of the system is [23]

$$U(t) = e^{-iH_0 t} e^{-iH_e t},$$
(2)

where  $H_0 = \sum_{j=1}^{2} \Omega(S_j^- + S_j^+)$ ,  $H_e$  is the effective Hamiltonian. In the large detuning  $\delta \gg \frac{1}{2}g$  and strong driving field  $2\Omega \gg \delta$ , g limit, the effective Hamiltonian for this interaction can be described as follows [23]:

$$H_{e} = \frac{1}{2}\lambda \left[ \sum_{j=1}^{2} (|e\rangle_{j} \langle e + |g\rangle_{j} \langle g) + \sum_{j,k=1, j \neq k}^{2} (S_{j}^{+}S_{k}^{+} + S_{j}^{+}S_{k}^{-}) + H.c \right],$$
(3)

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where  $\lambda = g^2/2\delta$ ,  $\delta$  is the detuning between  $\omega_0$  and  $\omega_a$ . From (3), we know that  $H_e$  is independent of creation and annihilation operators of the cavity mode and is only related with atomic operators. So the effects of cavity decay and thermal field are all eliminated. When two atoms exist in the cavity, after interaction time *t*, the state of the two atoms will undergo the following evolution:

$$|ee\rangle_{jk} \longrightarrow e^{-i\lambda t} [\cos \lambda t (\cos \Omega t |e\rangle_j - i \sin \Omega t |g\rangle_j) \times (\cos \Omega t |e\rangle_k - i \sin \Omega t |g\rangle_k) - i \sin \lambda t (\cos \Omega t |g\rangle_j - i \sin \Omega t |e\rangle_j) \times (\cos \Omega t |g\rangle_k - i \sin \Omega t |e\rangle_k)], \quad (4)$$

$$|eg\rangle_{jk} \longrightarrow e^{-i\lambda t} [\cos \lambda t (\cos \Omega t |e\rangle_j - i \sin \Omega t |g\rangle_j) \times (\cos \Omega t |g\rangle_k - i \sin \Omega t |e\rangle_k)$$

$$-i\sin\lambda t(\cos\Omega t|g\rangle_j - i\sin\Omega t|e\rangle_j) \times (\cos\Omega t|e\rangle_k - i\sin\Omega t|g\rangle_k)], \quad (5)$$

$$|ge\rangle_{jk} \longrightarrow e^{-i\lambda t} [\cos \lambda t (\cos \Omega t |g\rangle_j - i \sin \Omega t |e\rangle_j) \times (\cos \Omega t |e\rangle_k - i \sin \Omega t |g\rangle_k)$$

$$-i\sin\lambda t(\cos\Omega t|e\rangle_j - i\sin\Omega t|g\rangle_j) \times (\cos\Omega t|g\rangle_k - i\sin\Omega t|e\rangle_k)], \quad (6)$$

$$|gg\rangle_{jk} \longrightarrow e^{-i\lambda t} [\cos \lambda t (\cos \Omega t |g\rangle_j - i \sin \Omega t |e\rangle_j) \times (\cos \Omega t |g\rangle_k - i \sin \Omega t |e\rangle_k)$$

$$-i\sin\lambda t(\cos\Omega t|e\rangle_j - i\sin\Omega t|g\rangle_j) \times (\cos\Omega t|e\rangle_k - i\sin\Omega t|g\rangle_k)].$$
(7)

#### 3 The Controlled Teleportation of an Arbitrary Two-Atom Entangled State

Assume that the two atoms to be teleported are initially in the state

. . .

$$|\phi\rangle_{12} = a|gg\rangle_{12} + b|ge\rangle_{12} + c|eg\rangle_{12} + d|ee\rangle_{12}$$
(8)

where a, b, c and d are unknown coefficients,  $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$ .  $|e\rangle$  and  $|g\rangle$  are the excited and ground states of the atom. The quantum channels are a three-atom maximally entangled state and a two-atom maximally entangled state.

$$|\phi\rangle_{345} = \frac{1}{\sqrt{2}} (|ggg\rangle_{345} + i|eee\rangle_{345}), \tag{9}$$

$$|\phi\rangle_{67} = \frac{1}{\sqrt{2}} (|ge\rangle_{67} - i|eg\rangle_{67}).$$
(10)

Here the atoms 1, 2, 3 and 6 belong to the sender Alice, atoms 4, 7 belong to the receiver Bob, and atom 5 the controlling atom belongs to Charlie. The initial state of the whole system is given by

$$|\psi\rangle_{1234567} = \frac{1}{2} (a|gg\rangle_{12} + b|ge\rangle_{12} + c|eg\rangle_{12} + d|ee\rangle_{12}) \otimes (|ggg\rangle_{345} + i|eee\rangle_{345})$$
  
 
$$\otimes (|ge\rangle_{67} - i|eg\rangle_{67}).$$
(11)

Then Alice sends atoms 1, 3 into a single-mode cavity and atoms 2, 6 into another cavity. We only write the time evolution of the first term part  $|gggggge\rangle_{1234567}$  of (11).

$$|gggggge\rangle_{1234567} = e^{-2i\lambda t} [\cos \lambda t (\cos \Omega t |g\rangle_1 - i \sin \Omega t |e\rangle_1) (\cos \Omega t |g\rangle_3$$
$$- i \sin \Omega t |e\rangle_3) - i \sin \lambda t (\cos \Omega t |e\rangle_1 - i \sin \Omega t |g\rangle_1)$$
$$\times (\cos \Omega t |e\rangle_3 - i \sin \Omega t |g\rangle_3)] \times [\cos \lambda t (\cos \Omega t |g\rangle_2]$$

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$$- i \sin \Omega t |e\rangle_{2} (\cos \Omega t |g\rangle_{6} - i \sin \Omega t |e\rangle_{6}) - i \sin \lambda t$$

$$\times (\cos \Omega t |e\rangle_{2} - i \sin \Omega t |g\rangle_{2} )(\cos \Omega t |e\rangle_{6} - i \sin \Omega t |g\rangle_{6})]$$

$$\times |gge\rangle_{457}.$$
(12)

In the same method, we can choose  $\lambda t = \frac{1}{4}\pi$ ,  $\Omega t = \pi$  by modulating the driving field appropriately. The whole system will evolve into

$$\begin{split} |\psi\rangle &= \frac{1}{4} [|eeee\rangle_{1326} \times (-a|gge\rangle_{457} - b|ggg\rangle_{457} + c|eee\rangle_{457} + d|eeg\rangle_{457}) \\ &+ |eegg\rangle_{1326} \times (-ia|gge\rangle_{457} - ib|ggg\rangle_{457} + ic|eee\rangle_{457} - id|eeg\rangle_{457}) \\ &+ |ggee\rangle_{1326} \times (-ia|gge\rangle_{457} - b|ggg\rangle_{457} - ic|eee\rangle_{457} - id|eeg\rangle_{457}) \\ &+ |gggg\rangle_{1326} \times (a|gge\rangle_{457} - b|ggg\rangle_{457} + c|eee\rangle_{457} - d|eeg\rangle_{457}) \\ &+ |egge\rangle_{1326} \times (-a|eeg\rangle_{457} + b|eee\rangle_{457} - c|ggg\rangle_{457} + d|gge\rangle_{457}) \\ &+ |egge\rangle_{1326} \times (-ia|eeg\rangle_{457} - ib|eee\rangle_{457} - ic|ggg\rangle_{457} - id|gge\rangle_{457}) \\ &+ |geeg\rangle_{1326} \times (-ia|eeg\rangle_{457} + b|eee\rangle_{457} - ic|ggg\rangle_{457} - id|gge\rangle_{457}) \\ &+ |geeg\rangle_{1326} \times (-ia|eeg\rangle_{457} + b|eee\rangle_{457} - c|ggg\rangle_{457} - id|gge\rangle_{457}) \\ &+ |geeg\rangle_{1326} \times (a|eeg\rangle_{457} - b|gge\rangle_{457} - c|eeg\rangle_{457} - d|gge\rangle_{457}) \\ &+ |geeg\rangle_{1326} \times (a|eeg\rangle_{457} - b|gge\rangle_{457} - c|eeg\rangle_{457} + d|eee\rangle_{457}) \\ &+ |gege\rangle_{1326} \times (-a|ggg\rangle_{457} - b|gge\rangle_{457} - c|eeg\rangle_{457} + d|eee\rangle_{457}) \\ &+ |ggeg\rangle_{1326} \times (-a|ggg\rangle_{457} - b|gge\rangle_{457} - c|eeg\rangle_{457} - id|eee\rangle_{457}) \\ &+ |ggeg\rangle_{1326} \times (-a|ggg\rangle_{457} - b|gge\rangle_{457} - c|eeg\rangle_{457} - id|eee\rangle_{457}) \\ &+ |ggeg\rangle_{1326} \times (-a|ggg\rangle_{457} - ib|gge\rangle_{457} - ic|gge\rangle_{457} - id|eee\rangle_{457}) \\ &+ |ggeg\rangle_{1326} \times (-ia|ggg\rangle_{457} - ib|geg\rangle_{457} - ic|gge\rangle_{457} - id|ggg\rangle_{457}) \\ &+ |gee\rangle_{1326} \times (a|eee\rangle_{457} - ib|eeg\rangle_{457} - ic|gge\rangle_{457} - id|ggg\rangle_{457}) \\ &+ |gee\rangle_{1326} \times (a|eee\rangle_{457} - b|eeg\rangle_{457} - c|gge\rangle_{457} - d|ggg\rangle_{457}) \\ &+ |gee\rangle_{1326} \times (a|eee\rangle_{457} - b|eeg\rangle_{457} - c|gge\rangle_{457} - d|ggg\rangle_{457}) \\ &+ |gee\rangle_{1326} \times (a|eee\rangle_{457} - b|eeg\rangle_{457} - c|gge\rangle_{457} - d|ggg\rangle_{457}) \\ &+ |gee\rangle_{1326} \times (a|eee\rangle_{457} - b|eeg\rangle_{457} - c|gge\rangle_{457} - d|ggg\rangle_{457}) \\ &+ |gee\rangle_{1326} \times (a|eee\rangle_{457} - b|eeg\rangle_{457} - c|gge\rangle_{457} - d|ggg\rangle_{457}) \\ &+ |geg\rangle_{1326} \times (a|eee\rangle_{457} - b|eeg\rangle_{457} - c|gge\rangle_{457} - d|ggg\rangle_{457}) \\ &+ |gegg\rangle_{1326} \times (a|eee\rangle_{457} - b|eeg\rangle_{457} - c|gge\rangle_{457} - d|ggg\rangle_{457}) \\ &+ |gegg\rangle_{1326} \times (a|eee\rangle_{457} - b|eeg\rangle_{457} - c|gge\rangle_{457} - d|ggg\rangle_{457}) \\ &+ |gegg\rangle_{1326} \times (a|eee\rangle_{457} - b|eeg\rangle_{457} - c|$$

In order to realize the teleportation, Alice should make a separate measurement on atoms 1, 2, 3 and 6. If Alice detects the atoms in the state  $|eeee\rangle_{1326}$ , after measurement the state of atoms 4, 5 and 7 will collapse into

$$-a|gge\rangle_{457} - b|ggg\rangle_{457} + c|eee\rangle_{457} + d|eeg\rangle_{457}.$$
 (14)

Now Alice informs Bob and Charlie of the result of the measurement by the classical channels, after Charlie receives the information, if Charlie would like to help Bob with the teleportation, he should perform the Hadamard operation in the following forms on atom 5 in the basis  $|g\rangle$ ,  $|e\rangle$ .

$$H|g\rangle = \frac{1}{\sqrt{2}}(|g\rangle + |e\rangle), H|e\rangle = \frac{1}{\sqrt{2}}(|g\rangle - |e\rangle).$$
(15)

Equation (14) will become

$$-\frac{1}{\sqrt{2}}[|g\rangle_{5}(a|ge\rangle_{47}+b|gg\rangle_{47}-c|ee\rangle_{47}-d|eg\rangle_{47})$$

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$$+ |e\rangle_{5}(a|ge\rangle_{47} + b|gg\rangle_{47} + c|ee\rangle_{47} + d|eg\rangle_{47})].$$
(16)

If Charlie measures atom 5 in the  $|e\rangle_5$ , the atom 4 and 7 are in the state

$$a|ge\rangle_{47} + b|gg\rangle_{47} + c|ee\rangle_{47} + d|eg\rangle_{47}.$$
(17)

Then Charlie tells the measurement result to Bob via classical channel, Bob makes a unitary transformation  $I_4 \otimes \sigma_{7x}$  on atoms 4, 7 to recover the teleported state, (17) will become

$$a|gg\rangle_{47} + b|ge\rangle_{47} + c|eg\rangle_{47} + d|ee\rangle_{47}.$$
(18)

If Charlie measures atom 5 in the  $|g\rangle_5$ , the atoms 4 and 7 are in the state

$$a|ge\rangle_{47} + b|gg\rangle_{47} - c|ee\rangle_{47} - d|eg\rangle_{47}.$$
(19)

Then Bob makes a unitary transformation  $\sigma_{4z} \otimes \sigma_{7x}$  on atoms 4, 7, (19) will become

$$a|gg\rangle_{47} + b|ge\rangle_{47} + c|eg\rangle_{47} + d|ee\rangle_{47}.$$
(20)

Now the teleportation is successful. From (13) to (14), we can see that after Alice makes a separate measurement, quantum information of the teleported state is encoded into the state of the atoms 4, 5, 7, which is shared between Bob and Charlie, so Bob cannot fully recover the origin state. However, the receiver can successfully get access to the original state if Charlie collaborates through the local operation and classical communication with the receiver. From (13), we can see that when atoms 1, 3 and atoms 2, 6 enter into the cavity, because every atom has two levels, 16 kinds of different separate states can be derived. It is evident that Bob must operate relevant unitary transformation against Alice's and Charlie's different measurement results. By the similar method, if the measurement results of Alice are  $|eegg\rangle_{1326}$ ,  $|ggeg\rangle_{1326}$ ,  $|gggg\rangle_{1326}$ ,  $|eggg\rangle_{1326}$ ,  $|egee\rangle_{1326}$ ,  $|geeg\rangle_{1326}$ ,  $|geeg\rangle_{1326}$ ,  $|eegg\rangle_{1326}$ ,  $|eegg\rangle_{1326}$ ,  $|ggeg\rangle_{1326}$ ,  $|eggg\rangle_{1326}$ ,  $|egeg\rangle_{1326}$ ,  $|geeg\rangle_{1326}$ ,  $|eegg\rangle_{1326}$ ,  $|eegg\rangle_{1326}$ ,  $|eegg\rangle_{1326}$ ,  $|eegg\rangle_{1326}$ ,  $|eegg\rangle_{1326}$ ,  $|geeg\rangle_{1326}$ ,  $|eegg\rangle_{1326}$ ,  $|eegg\rangle_{132$ 

Now we calculate the probability of successful teleportation in this scheme. From (13) to (14), the probability of detecting the state  $|eeee\rangle_{1326}$  is 1/16, from (14) to (20), the probability of successful teleportation is 1.0. So we can know that the probability of successful teleportation is 1/16. The probabilities of successful teleportation in the other fifteen measurement results are easily derived. That is to say, the total success probability is  $1/16 \times 16 = 1.0$ .

Let us provide thorough physical analysis and discussions of this scheme. Firstly let the atoms with Alice interact in a driven cavity QED. The separate state of the two atoms may evolve into a two-atom maximally entangled state by properly choosing the time and coupling constants. The separate measurements on the atoms in a driven cavity QED substitute apparent (or direct) Bell measurements. Therefore the difficult apparent (or direct) Bell state measurements that Alice should perform in order to teleport his qubits are not needed. Secondly after Charlie performs the Hadamard operation on her atom, he can obtain two outcomes, if Charlie tells the measurement result to Bob via classical channel, Bob only makes an unitary transformation to recover the teleported state based on Charlie's measurement result.

Alice	Charlie	Bob operation	Alice	Charlie	Bob operation
eeee>1326	$ e\rangle_5$	$I_4 \otimes \sigma_{7x}$	$ eeeg\rangle_{1326}$	$ e\rangle_5$	$I_4 \otimes \sigma_{7z}$
$ eeee\rangle_{1326}$	$ g\rangle_5$	$\sigma_{4z} \otimes \sigma_{7x}$	$ eeeg\rangle_{1326}$	$ g\rangle_5$	$\sigma_{4z}\otimes\sigma_{7z}$
$ egee\rangle_{1326}$	$ e\rangle_5$	$U_4 \otimes \sigma_{7x}$	$ egeg\rangle_{1326}$	$ e\rangle_5$	$U_4\otimes\sigma_{7z}$
$ egee\rangle_{1326}$	$ g\rangle_5$	$\sigma_{4x}\otimes\sigma_{7x}$	$ egeg\rangle_{1326}$	$ g\rangle_5$	$\sigma_{4x} \otimes \sigma_{7z}$
$ eegg\rangle_{1326}$	$ e\rangle_5$	$I_4 \otimes U_7$	$ eege\rangle_{1326}$	$ e\rangle_5$	$I_4 \otimes I_7$
$ eegg\rangle_{1326}$	$ g\rangle_5$	$\sigma_{4z}\otimes U_7$	$ eege\rangle_{1326}$	$ g\rangle_5$	$\sigma_{4z}\otimes I_7$
$ geee\rangle_{1326}$	$ e\rangle_5$	$\sigma_{4x}\otimes\sigma_{7x}$	$ egge\rangle_{1326}$	$ e\rangle_5$	$U_4 \otimes I_7$
$ geee\rangle_{1326}$	$ g\rangle_5$	$U_4 \otimes \sigma_{7x}$	$ egge\rangle_{1326}$	$ g\rangle_5$	$\sigma_{4x} \otimes I_7$
<i>gggg</i> ) <sub>1326</sub>	$ e\rangle_5$	$\sigma_{4z}\otimes U_7$	$ ggeg\rangle_{1326}$	$ e\rangle_5$	$\sigma_{4z} \otimes \sigma_{7z}$
<i>gggg</i> ) <sub>1326</sub>	$ g\rangle_5$	$I_4 \otimes U_7$	$ ggeg\rangle_{1326}$	$ g\rangle_5$	$I_4 \otimes \sigma_{7z}$
$ eggg\rangle_{1326}$	$ e\rangle_5$	$U_4 \otimes U_7$	$ geeg\rangle_{1326}$	$ e\rangle_5$	$\sigma_{4x}\otimes\sigma_{7z}$
$ eggg\rangle_{1326}$	$ g\rangle_5$	$\sigma_{4x}\otimes U_7$	$ geeg\rangle_{1326}$	$ g\rangle_5$	$U_4\otimes\sigma_{7z}$
$ ggee\rangle_{1326}$	$ e\rangle_5$	$\sigma_{4z} \otimes \sigma_{7x}$	$ ggge\rangle_{1326}$	$ e\rangle_5$	$\sigma_{4z}\otimes I_7$
$ ggee\rangle_{1326}$	$ g\rangle_5$	$I_4 \otimes \sigma_{7x}$	$ ggge\rangle_{1326}$	$ g\rangle_5$	$I_4 \otimes I_7$
$ gegg\rangle_{1326}$	$ e\rangle_5$	$\sigma_{4x}\otimes U_7$	$ gege\rangle_{1326}$	$ e\rangle_5$	$\sigma_{4x} \otimes I_7$
$ gegg\rangle_{1326}$	$ g\rangle_5$	$U_4 \otimes U_7$	$ gege\rangle_{1326}$	$ g\rangle_5$	$U_4 \otimes I_7$

**Table 1** The results and the unitary transformations to finish the controlled teleportation of an arbitrary twoatom entangled state. *I* is the identity operator and  $\sigma_{x,y,z}$  are the usual Pauli matrices.  $U_i = |g\rangle_i \langle e| - |e\rangle_i \langle g|$ 

From the above analysis, we can see that Alice only needs to make a separate measurement, therefore the difficult apparent (or direct) Bell state measurements that Alice should perform in order to teleport her qubits are not needed. The two qubit state can be perfectly teleported with the help of Charlie's Hadamard operation. That is to say, after Charlie receives this information, if Charlie would like to help Bob with the teleportation, he should make an operation on atom 5. That is the controlled teleportation of an arbitrary two-atom state. All the above we suppose a three-qubit GHZ state as the quantum channel, only a atom is supposed as the controlling agent. In the same method, if we generalized a (n + 2)-qubit GHZ state, two qubits of (n + 2)-qubit GHZ state belong to Alice and Bob, respectively, while the other *n* GHZ qubits belong to *n* agents, we can successfully get the arbitrary two-atom two-level entangled state as long as all the agents cooperate and send classical communication. Therefore the controlled teleportation is useful in networked quantum information processing.

Finally, it is necessary to give a brief discussion on the experimental matters. We consider the typical experimental values of the parameters for Rydberg atoms with principal quantum numbers 49, 50, 51, the radiative time is about  $T_r = 3 \times 10^{-2}$  s, and the coupling constant is  $g = 2\pi \times 24$  kHz. For a normal cavity, the decay time can reach  $T_c = 1.0 \times 10^{-3}$  s. Then we get that the interaction time of atom and cavity is on the order of  $10^{-4}$  s. Hence, the total time for the whole system is much shorter than  $T_r$  and  $T_c$ , so that the present scheme might be realizable based on cavity QED techniques.

#### 4 Summary

We have proposed a simple scheme to realize the controlled teleportation of an arbitrary two-atom state  $|\phi\rangle_{12} = a|gg\rangle_{12} + b|ge\rangle_{12} + c|eg\rangle_{12} + d|ee\rangle_{12}$  in driven cavity QED. Two

atoms exist in single-mode cavity and are driven by a classical field. By detecting the states of atoms, we can achieve the controlled teleportation. The scheme is insensitive to the cavity decay and the thermal field. Meanwhile this idea can be used to teleport an arbitrary two-atom state from Alice to a receiver Bob via the control of n agents. The probability of the success can reach 1.0. Two-atom maximally entangled state can be readily prepared by atom-cavity field interaction, which has been experimentally realized [24]. Thus this scheme is realizable with technique presently available.

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